Analysis of orthotropic beams using model order reduction

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Raul Rubio Serrano*, Àlex Ferrer Ferré, Joaquín Hernández Ortega, Xavier Martínez Garcia

rrubio@cimne.upc.edu

CIMNE

EXCELENCIA SEVERO OCHOA

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MOTIVATION

BROM

CASE STUDY

Motivation

Simulation of beams made of composite materials

Isotropic materials
- FE ✓
- Beam theory ✓

Orthotropic materials
- FE ✓
- Beam theory ✗

bROM: new beam model for isotropic and orthotropic materials

?

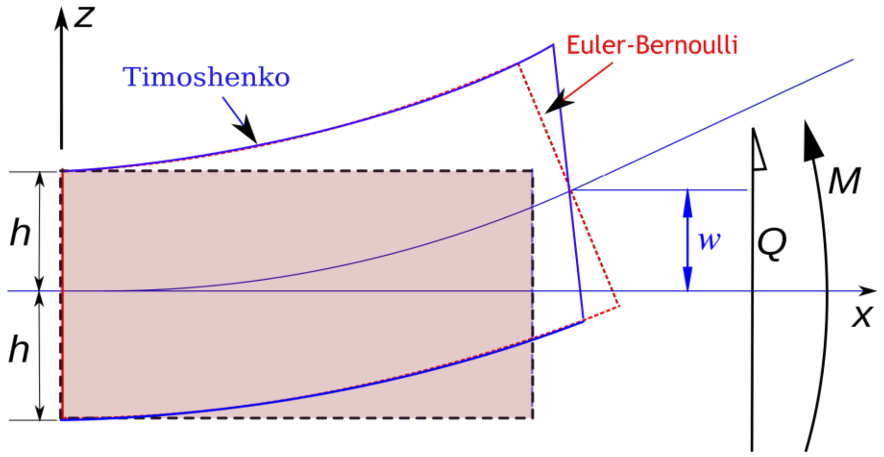
FE versatile but computationally expensive

ROM: faster solutions able to reproduce FE

Beam theory

6 degrees of freedom: 3 translations and 3 rotations

Analytical dependency with beam length



[1]

- Euler-Bernoulli

$$k_a = \frac{EA}{L_e}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad k_t = \frac{GJ}{L_e}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad k_b = \frac{EI}{L_e^3}\left[\begin{array}{cc|cc} 12 & -12 & 6L_e & 6L_e \\ -12 & 12 & -6L_e & -6L_e \\ \hline 6L_e & -6L_e & 4L_e^2 & 2L_e^2 \\ 6L_e & -6L_e & 2L_e^2 & 4L_e^2 \end{array}\right]$$

$$k = \begin{bmatrix} k_a & 0 & 0 & 0 \\ 0 & k_b^z & 0 & 0 \\ 0 & 0 & k_b^y & 0 \\ 0 & 0 & 0 & k_t \end{bmatrix}$$

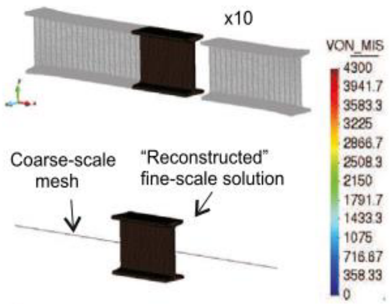
- Timoshenko

$$k_b = \frac{EI}{L_e^3}\left[\begin{array}{cc|cc} 12\phi & -12\phi & 6L_e\phi & 6L_e\phi \\ -12\phi & 12\phi & -6L_e\phi & -6L_e\phi \\ \hline 6L_e\phi & -6L_e\phi & (4+\phi)\bar{\phi}L_e^2 & (2-\phi)\bar{\phi}L_e^2 \\ 6L_e\phi & -6L_e\phi & (2-\phi)\bar{\phi}L_e^2 & (4+\phi)\bar{\phi}L_e^2 \end{array}\right]$$

$$\bar{\phi} = \frac{1}{1+\phi} \quad , \quad \phi = \frac{12EI}{kGAL_e^2}$$

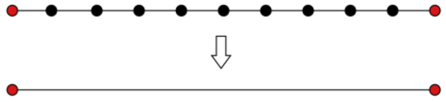
BROM

- ROM

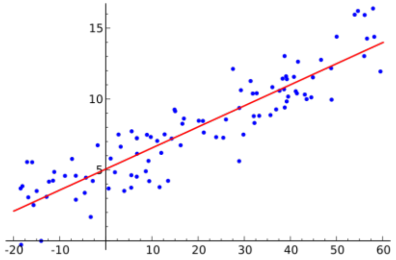


[2]

- Condensation



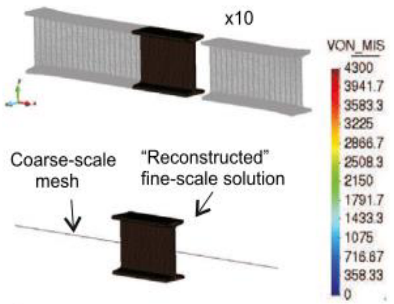
- Regression



[3]

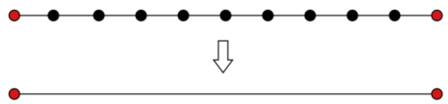
BROM

- **ROM**

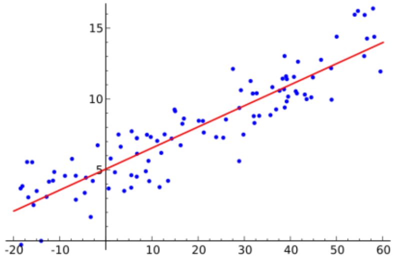


[2]

- Condensation



- Regression

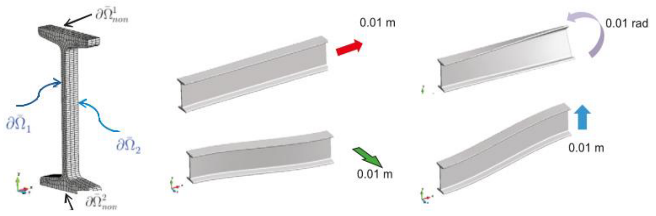


[3]

Multiscale method for periodic structures using domain decomposition and ECM-hyper reduction [2]

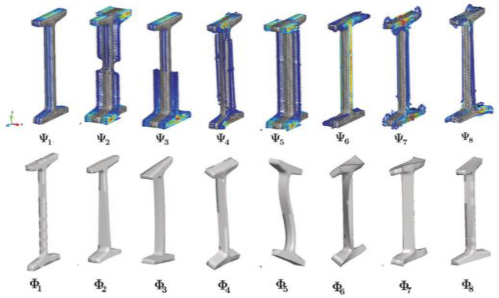
Offline stage

Training (FE)



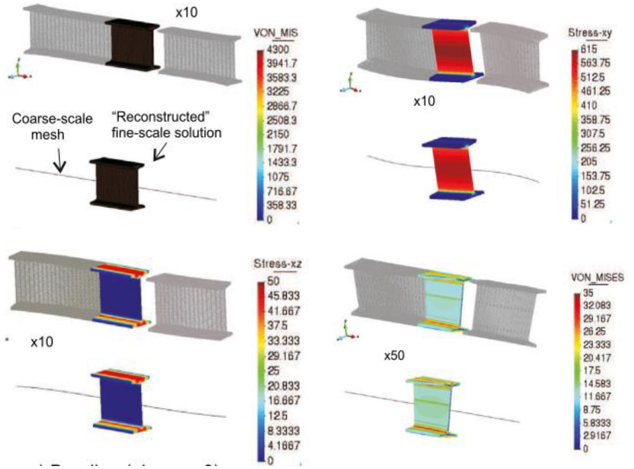
Provide reliable data of the problem

Model order reduction



Find a reduced parameterization of the data

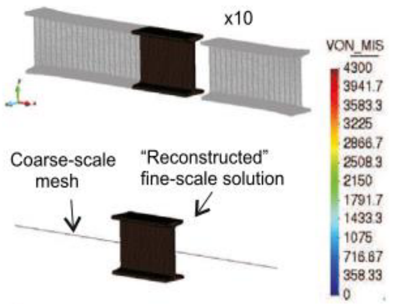
Online stage



Solve the ROM and reconstruct 3D solution

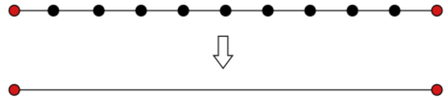
BROM

- ROM

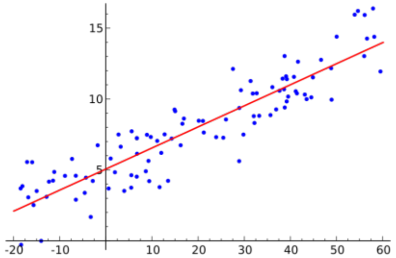


[2]

- **Condensation**



- Regression



[3]

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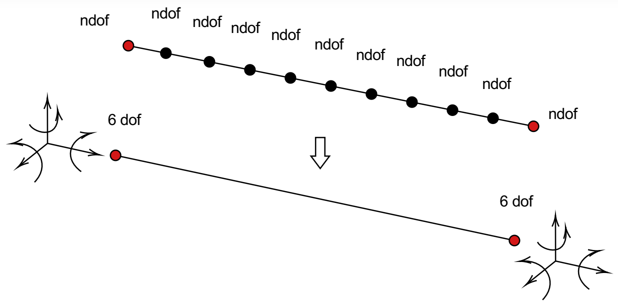
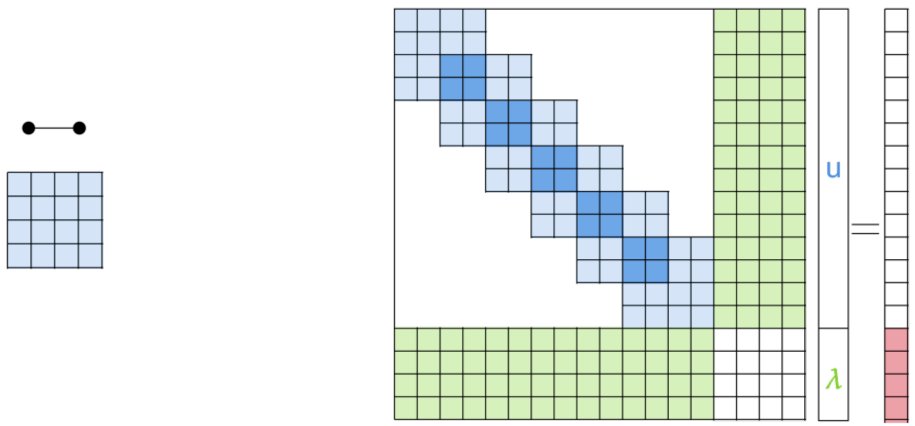
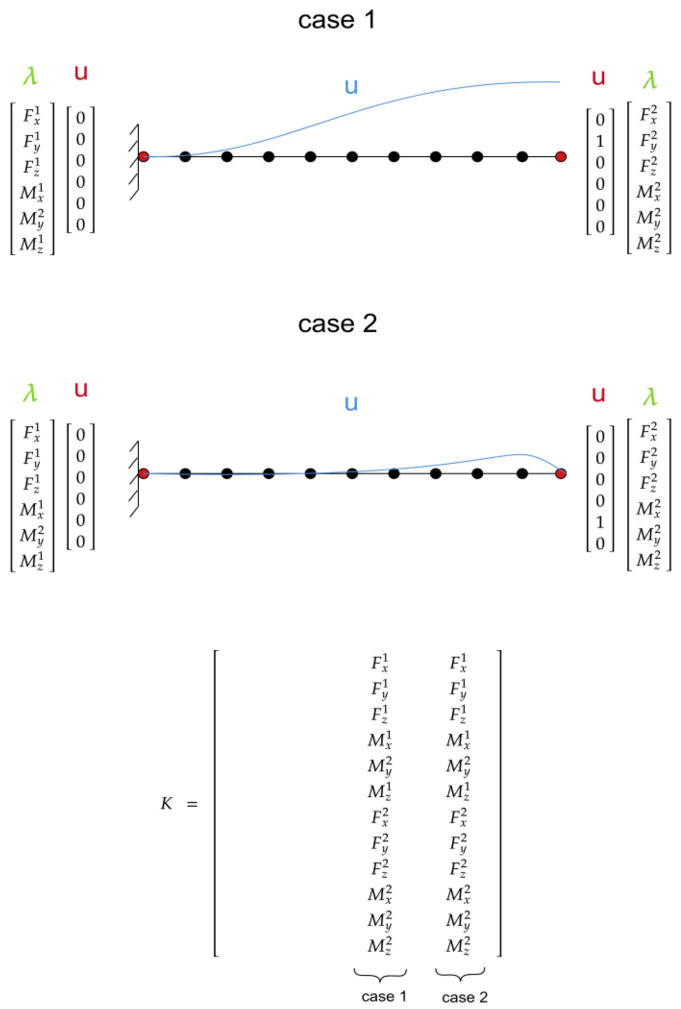
Condensation



Assemble elemental matrix and apply perturbations

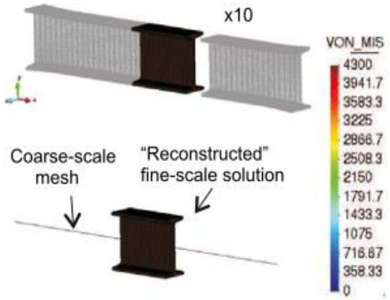


Apply perturbations to rigid body modes



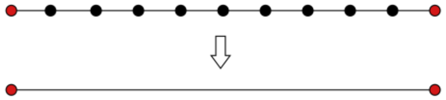
BROM

- ROM

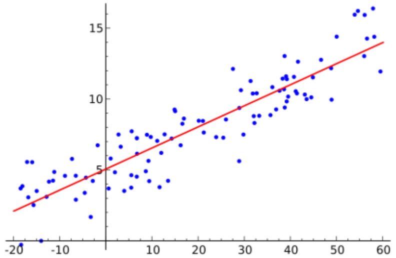


[2]

- Condensation

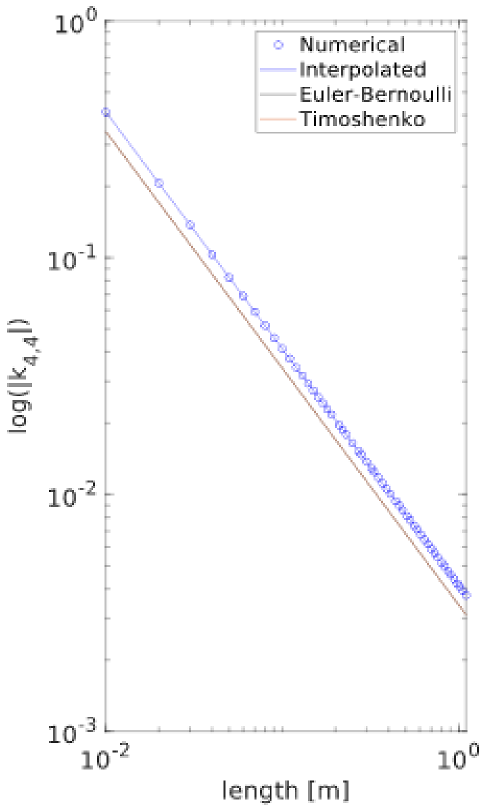


- **Regression**



[3]

Regression



Linear interpolation with local support

$$K_{ij}(l) = N_1^e(l) K_1^e + N_2^e(l) K_2^e$$

$$N_1^e(l) = \begin{cases} N_1^e = (1-\eta) \text{ where } \eta = \frac{l-L_i}{L_{i+1}-L_i} & \forall l \in [L_i, L_{i+1}] \\ 0 \text{ otherwise} \end{cases}$$

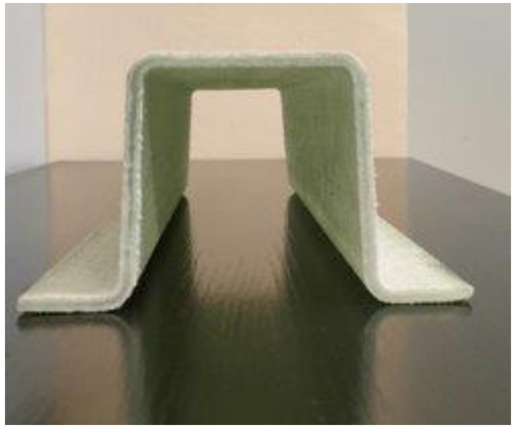
$$N_2^e(l) = \begin{cases} N_2^e = \eta & \forall l \in [L_i, L_{i+1}] \\ 0 \text{ otherwise} \end{cases}$$

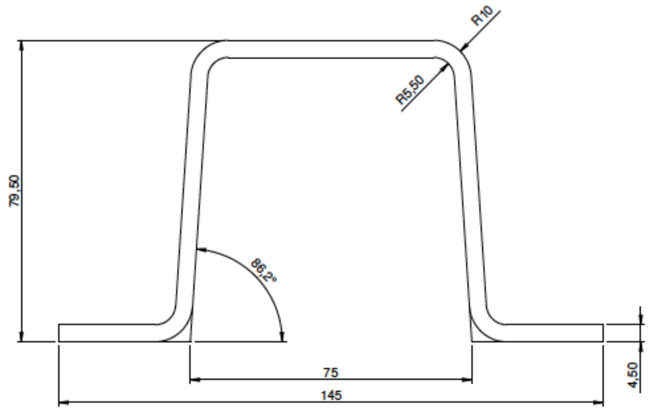
- Stable elements
- Error approaches to 0 at the end of the segment

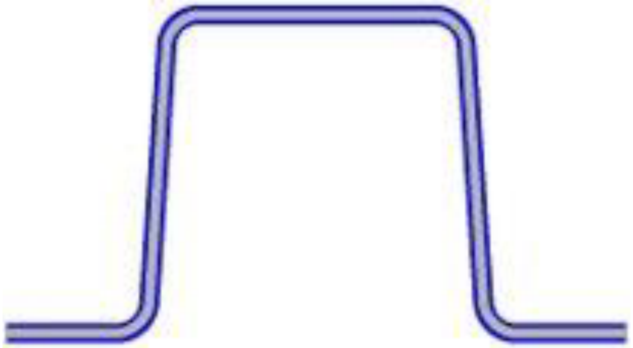
- Not as easy implementation

CASE STUDY

Geometry and Material

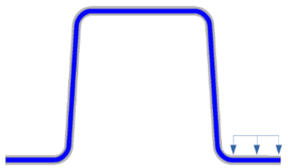


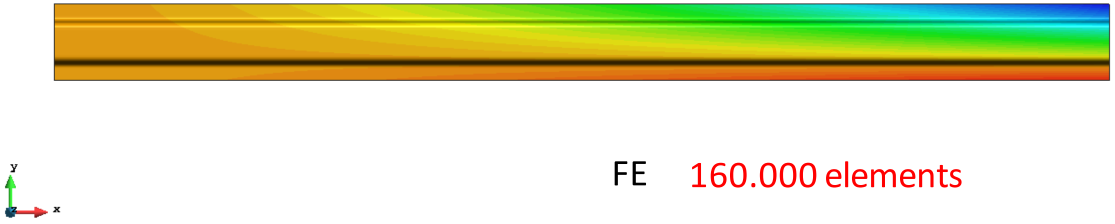




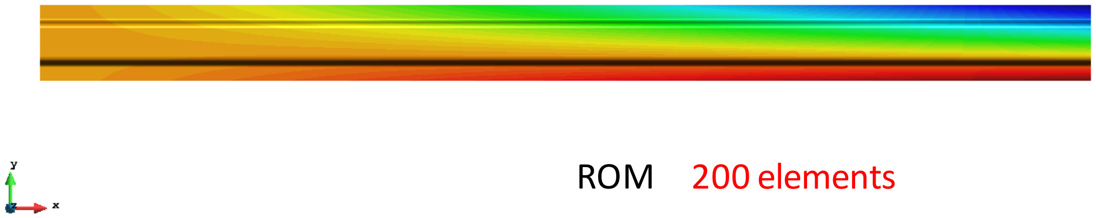
	Laminate	Unidirectional
E1	20.2 GPa	46.7 GPa
E2	18.5 GPa	7.75 GPa
E3	7.33 GPa	7.75 GPa
G1	7.33 GPa	2.46 GPa
G2	2.32 GPa	2.46 GPa
G3	2.32 GPa	2.46 GPa
ν1	0.299	0.0466
ν2	0.0466	0.0466
ν3	0.0466	0.0466

ROM vs FE

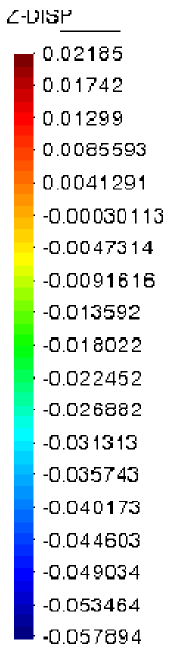


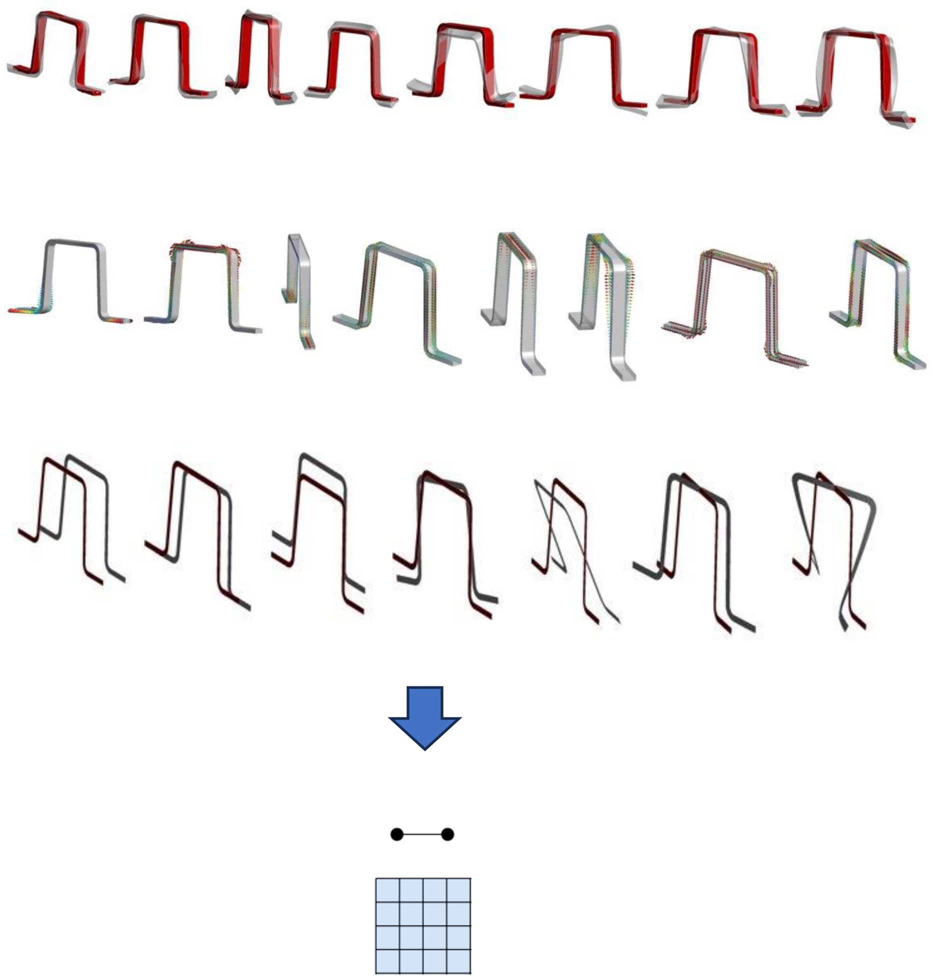


FE 160.000 elements



ROM 200 elements





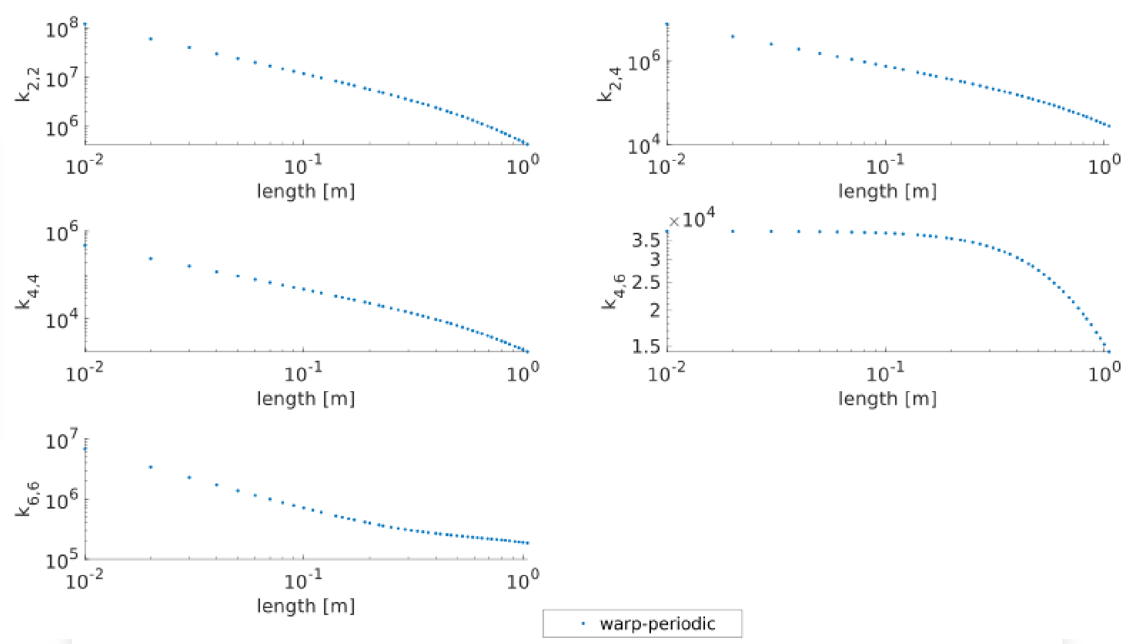
Condensation

Coupling

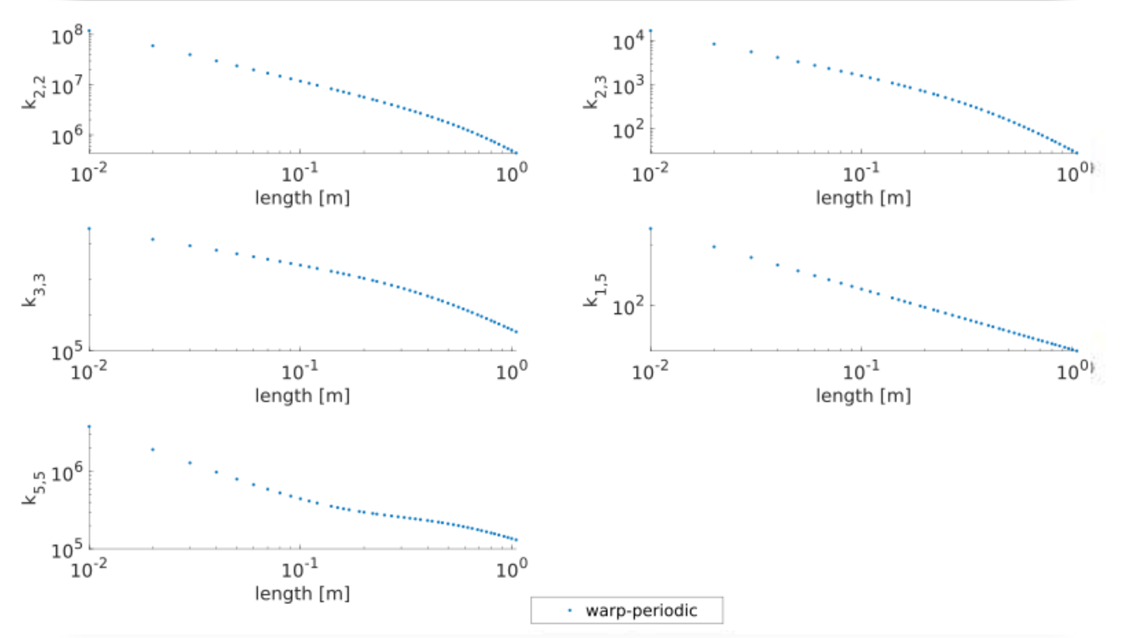
- Geometrical: due to non-symmetry of the cross section

- Material: due to the orthotropy of the material

Run with isotropic material to spot geometrical coupling



Material coupling cannot be captured with classical theories but it can be with the bROM



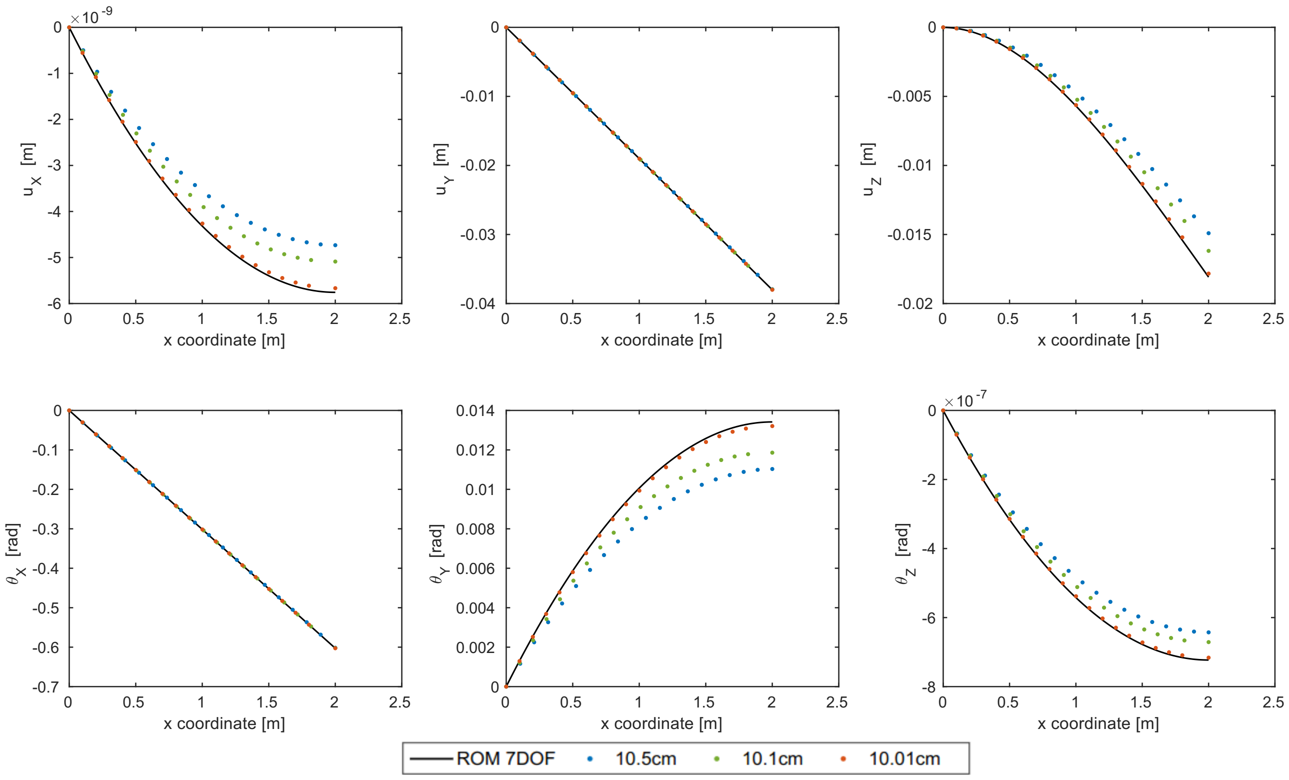
Validation

- Comparison bROM vs ROM
- Element size influence

?

- Stable solutions
- bROM can reproduce ROM solutions

✓



Conclusions

Stable elements

Capable of reproducing full ROM solutions

Orthotropy captured

bROM a method for analyzing composite beams in the traditional way

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References

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[2] J. A. Hernández, A multiscale method for periodic structures using domain decomposition and ecm-hyperreduction, Computer Methods in Applied Mechanics and Engineering 368 (2020) 113–192.
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[3] https://en.wikipedia.org/wiki/Regression_analysis